CALCULATION OF TRANSONIC NOZZLE FLOW

by JOSEPH L. SIMS
Aero-Astrodynamics Laboratory

NASA

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Joseph L. Sims

FLUID DYNAMICS RESEARCH OFFICE
AERODYNAMICS DIVISION
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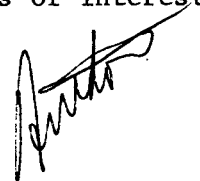
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Joseph L. Sims

George C. Marshall Space Flight Center

Huntsville, Alabama

ABSTRACT

An approximate solution of the transonic throat flow in a DeLaval nozzle is found by expanding the potential function in a power series about the critical line. Five terms were used in the present series expansion, and the complete potential flow equation of motion was used.

Solutions of the present set of equations are functions of two independent parameters: the radius of curvature of the nozzle wall and the ratio of specific heats of the fluid medium. The solution of the resultant equations is complex enough to make an electronic computer program desirable. For this reason, basic results of a series of solutions over a wide range of the two independent parameters are given in tabular form. From these tabulated results, any quantities of interest in the flow field may be rapidly computed.

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DEFINITION OF SYMBOLS

Symbol	Definition
a	local speed of sound
a^*	critical speed of sound
x, r, $\emptyset$	axial, radial, and meridional coordinates of the cylindrical coordinate system. The origin is located at the point the critical line crosses the longitudinal axis of the nozzle.
$\tilde{u}$	velocity component in x-direction
$\tilde{v}$	velocity component in r-direction
U	dimensionless velocity component in r-direction $(U = \frac{\tilde{u}}{a^*})$
V	dimensionless velocity component in r-direction $(V = \frac{\tilde{U}}{a^*})$
u	dimensionless perturbation velocity in x-direction
v	dimensionless perturbation velocity in r-direction
r_s	radial coordinate of intersection of critical line with nozzle wall
x_s	axial coordinate of intersection of critical line with nozzle wall
α	velocity gradient on nozzle center line at the critical line
ϵ	distance from vertex of the critical line

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SUMMARY

An approximate solution of the transonic throat flow in a De Laval nozzle is found by expanding the potential function in a power series about the critical line. Five terms were used in the present series expansion, and the complete potential flow equation of motion was used.

Solutions of the present set of equations are functions of two independent parameters: the radius of curvature of the nozzle wall and the ratio of specific heats of the fluid medium. The solution of the resultant equations is complex enough to make an electronic computer program desirable. For this reason, basic results of a series of solutions over a wide range of the two independent parameters are given in tabular form. From these tabulated results, any quantities of interest in the flow field may be rapidly computed.

I. INTRODUCTION

Transonic flow through the throat of an axially symmetric De Laval nozzle has, in general, been solved only by assuming the potential function to be given by a power series and obtaining the solution through numerical analysis. Two of the most generally available solutions are contained in References 1 (Oswatitsh and Rothstein) and 2 (Sauer). Although the general approach to the problem in these two papers is very similar, the final methods of solution are quite different.

In the method of Oswatitsh and Rothstein, it is necessary to perform several iterations on the basic solution to obtain an essentially closed solution for small radius of curvature at the throat. This process is tedious and, since numerical differentiation procedures are required in the iterations, it may not be stable for the number of iterations required to obtain a closed solution. On the other hand, Sauer's method uses only two terms in the series expansion about the critical line and should be used only for nozzles with a large radius of curvature.

The present paper consists of an extension of the Sauer method by using more terms of the series expansion about the critical line. This should increase the accuracy of the solution, especially for smaller values of the throat radius of curvature. Unfortunately, the coefficients of the series expansion became so complex that it is impractical to attempt the derivation of a large number of the coefficients. Therefore, the solution given in this paper is still not a closed series solution for radii of curvature as small as those encountered in typical present-day propulsion nozzles.

II. EQUATION OF MOTION

A cylindrical $(x, r, \emptyset)$ coordinate system is used with the x-axis along the centerline of the nozzle with the origin at the position where the critical velocity line crosses the nozzle centerline. Assuming irrotational flow of a perfect, non-heat-conducting fluid with a constant ratio of specific heats, the potential equation of motion is

$$\left(1 - \frac{\tilde{u}^2}{a^2}\right) \frac{\partial \tilde{u}}{\partial x} + \left(1 - \frac{\tilde{v}^2}{a^2}\right) \frac{\partial \tilde{v}}{\partial r} - 2 \frac{\tilde{u}\tilde{v}}{a^2} \frac{\partial \tilde{u}}{\partial r} + \frac{\tilde{u}}{r} = 0. \tag{1}$$

The local sonic velocity may be related to the critical sonic velocity, a^*, by

$$a^2 = \frac{\gamma + 1}{2} a^{*2} = \frac{\gamma - 1}{2} (\tilde{u}^2 + \tilde{v}^2). \tag{2}$$

Now, substituting (2) into (1) and letting

$$U = \tilde{u}/a^*, \quad V = \tilde{v}/a^*, \tag{3}$$

the potential equation becomes

$$\frac{\partial U}{\partial x} \left(1 - U^2 - \frac{\gamma - 1}{\gamma + 1} V^2\right) + \frac{\partial V}{\partial r} \left(1 - V^2 - \frac{\gamma - 1}{\gamma + 1} U^2\right)$$

$$- \frac{4}{\gamma + 1} UV \frac{\partial U}{\partial r} + 1 - \frac{\gamma - 1}{\gamma + 1} (U^2 + V^2) \frac{V}{r} = 0. \tag{4}$$

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If equation (4) is limited to a small region in the immediate vicinity of the critical curve, it is permissible to set

$$U = 1 + u, \quad V = v, \tag{5}$$

where u and v are small quantities. Upon substitution of (5) into (4), there results

$$\frac{\partial u}{\partial x} \left(u^2 + 2u + \frac{\gamma - 1}{\gamma + 1} v^2 \right) + \frac{\partial v}{\partial r} \left[v^2 + \frac{\gamma - 1}{\gamma + 1} (u^2 + 2u) - \frac{2}{\gamma + 1} \right]$$

$$+ \frac{4}{\gamma - 1} (1 + u) \, v \, \frac{\partial u}{\partial r} + \frac{v}{r} \left[\frac{\gamma - 1}{\gamma + 1} (u^2 + 2u + v^2) - \frac{2}{\gamma + 1} \right] = 0, \tag{6}$$

which is the equation for which a solution will be sought.

III. METHOD OF SOLUTION

Considering the symmetry about the x-axis, it is assumed that a potential function satisfying equation (6) can be found in the following form:

$$\Phi = f_o(x) + r^2 f_2(x) + r^4 f_4(x) + r^6 f_6(x) + r^8 f_8(x) + \dots \tag{7}$$

Then

$$u = \frac{\partial \Phi}{\partial x} = f_o'(x) + r^2 f_2'(x) + r^4 f_4'(x) + r^6 f_6'(x) + r^8 f_8'(x) + \dots \tag{8}$$

$$v = \frac{\partial \Phi}{\partial y} = 2r f_2(x) + 4r^3 f_4(x) + 6r^5 f_6(x) + 8r^7 f_8(x) + \dots,$$

where primes denote derivatives with respect to x. When equations (8) and the required partial derivatives of equations (8) are substituted

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into equation (6), the resulting equation can be arranged in powers of r. Then the individual coefficients of the powers of r are equated to zero and the following equations are obtained.

$$\frac{8}{\gamma + 1}\left[1 - \frac{\gamma - 1}{2} f'_o(2 + f'_o)\right] f_2 = f''_o \, f'_o(2 + f'_o) \tag{9}$$

$$\frac{32}{\gamma + 1}\left[1 - \frac{\gamma - 1}{2} f'_o(2 + f'_o)\right] f_4 = 2f''_o \, f'_2(1 + f'_o) + f''_2 \, f'_o(2 + f'_o) \tag{10}$$

$$+ 4\frac{\gamma - 1}{\gamma + 1} f''_o \, f_2{}^2 + 8f_2 f'_2(1 + f'_o) + 16\frac{\gamma}{\gamma + 1} f_2{}^3$$

$$\frac{72}{\gamma + 1}\left[1 - \frac{\gamma - 1}{2} f'_o \,(2 + f'_o)\right] f_6 = f''_o \left[2f'_4(1 + f'_o) + f'_2{}^2\right] + 2f''_2 f'_2(1 + f'_o)$$

$$+ f''_4 f'_o(2 + f'_o) + 4\frac{\gamma - 1}{\gamma + 1} (4f''_o f_2 f_4 + f''_2 f_2{}^2)$$

$$+ 2\frac{\gamma - 1}{\gamma + 1}\left(2 \, [2f'_4(1 + f'_o) + f'_2{}^2]f_2 + 16 \, f'_2 f_4(1 + f'_o)\right) \tag{11}$$

$$+ 80f_2{}^2 \, f_4 + \frac{16}{\gamma + 1}\left[2(f_2 f'_4 + f_4 f'_2) \, (1 + f'_o) + f_2 f'_2{}^2\right] + 48\frac{\gamma - 1}{\gamma + 1} f_2{}^2 f_4.$$

$$\frac{128}{\gamma + 1}\left[1 - \frac{\gamma - 1}{2} f'_o \,(2 + f'_o)\right] f_8 = 2f''_o \, f'_6 \, (1' + f'_o) + 2f''_o f'_2 \, f'_4$$

$$+ 2f''_2 f'_4 \, (1 + f'_o) + f''_2 f'_2{}^2 + 2f''_4 f'_2 \, (1 + f'_o) \tag{12}$$

$$+ f''_6 f'_6(2 + f'_o) + 4\frac{\gamma - 1}{\gamma + 1}\left\{2f_2 f'_6(1 + f'_o)\right.$$

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(equation 12 continued)

$$+ 2f_2f_2'f_4' + 8f_4f_4'(1 + f_o') + 4f_4f_2'^2$$

$$+ 18f_6f_2'(1 + f_o') + 6f_2f_6(f_o'' + 3f_2) + 4f_4^2 (f_o'' + 6f_2)$$

$$+ 4f_2''f_2f_4 + f_4''f_2^2 \Big\} + \frac{16}{\gamma + 1} \; 3f_2f_6'(1 + f_o') + 4f_4f_4'(1 + f_o')$$

$$+ 3f_6f_2'(1 + f_o') + f_2'(3f_2f_4' + 2f_2'f_4).$$

Now, equations (9) through (12) express all coefficients, in order, by the function $f_o'(x)$ and its derivatives. The function

$$f_o'(x) = u_o(x)$$

is the velocity along the nozzle centerline, and once it is determined, the solution of equations (9) through (12) for the higher coefficients may be immediately obtained. If we assume $u_o(x)$ to be a linear function, close to the critical line, then

$$f_o' = u_o(x) = \alpha x, \tag{13}$$

where α is an unspecified constant.

When equation (13) is inserted into equations (9) through (12), we obtain

$$f_2 = \frac{\gamma + 1}{8} \left\{ 2\alpha^2 x + (2\gamma - 1) \; \alpha^3 x^2 + 2\gamma(\gamma - 1) \; \alpha^4 x^3 + \right.$$

$$\left. + \frac{\gamma - 1}{2} \; (4\gamma^2 - 2\gamma - 1) \; \alpha^5 x^4 + \dots \right\}. \tag{14}$$

$$f_4 = \frac{1}{2} \left(\frac{\gamma + 1}{8} \right)^2 \left\{ 2\alpha^3 + 2(6\gamma - 1) \alpha^4 x + (36\gamma^2 - 19\gamma - 4)\alpha^5 x^2 \right.$$

$$\left. + 2(40\gamma^3 - 38\gamma^2 - 3\gamma + 4) \alpha^6 x^3 + \ldots \right\} . \tag{15}$$

$$f_6 = \frac{2}{3} \left(\frac{\gamma + 1}{8} \right)^3 \left\{ (6\gamma - 1) \alpha^5 + (56\gamma^2 - 20\gamma - 3) \alpha^6 x + \ldots \right\}. \tag{16}$$

$$f_8 = \frac{1}{24} \left(\frac{\gamma + 1}{8} \right)^4 \left\{ (544\gamma^2 - 151\gamma - 33) \alpha^7 + \ldots \right\} . \tag{17}$$

Now α and x are of the same order of magnitude so that the product $\alpha^p x^q$ is the same order of magnitude as α^{p+q}. If equations (7) and equations (14) through (17) are to be of the same order, f_8 must be restricted to a single term (α^7) and f_2 to f_6 will be restricted to terms of this same $(\alpha^p x^q \approx \alpha^7)$ order.

To solve for α, the streamline adjacent to the nozzle wall must have the same curvature as the nozzle wall. Thus,

$$\frac{1}{\rho} = \frac{1}{1 + u} \frac{\partial v}{\partial x} , \tag{18}$$

where u and $\partial v/\partial x$ are evaluated at the intersection of the nozzle wall and the critical velocity line. Inserting equations (8) and (14) through (17) into (18), results in

$$A\alpha^2 + B\alpha^3 + C\alpha^4 + E\alpha^5 + G\alpha^6 = \frac{1}{\rho_s} , \tag{19}$$

where

$$A = \frac{\gamma + 1}{2} r_s , \tag{20}$$

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$$B = (\gamma^2 - 1) \, x_s r_s, \tag{21}$$

$$C = \frac{\gamma + 1}{16} \left[8(3\gamma^2 - 5\gamma + 2) \, x_s^2 + 3(2\gamma^2 + \gamma - 1) \, r_s^2 \right] r_s, \tag{22}$$

$$E = \left(\frac{\gamma + 1}{4} \right)^2 (36\gamma^2 - 33\gamma + 5) \, r_s^3 \, x_s, \tag{23}$$

and

$$G = \left(\frac{\gamma + 1}{4} \right)^3 (28\gamma^2 - 19\gamma + 2) \, r_s^5. \tag{24}$$

The origin of the x-axis is still an unknown for which a solution must be found. If ϵ is defined as the distance from the point of intersection of the critical line, and the x-axis back to the throat section, then ϵ can be computed from the requirement that $v = 0$ at the throat wall. Then,

$$H\epsilon^3 + K\epsilon^2 + L\epsilon + M = 0, \tag{25}$$

where

$$H = 2\gamma(\gamma - 1) \, \alpha^2 \tag{26}$$

$$K = \left[2\gamma - 1 + \frac{\gamma + 1}{8} (36\gamma^2 - 19\gamma - 4) \, \alpha^2 \right] \alpha \tag{27}$$

$$L = 2 + \frac{\gamma + 1}{4} (6\gamma - 1) \, \alpha^2 + 2 \left(\frac{\gamma + 1}{8} \right)^2$$

$$\cdot \, (56\gamma^2 - 20\gamma - 3) \, \alpha^4 \tag{28}$$

and

$$M = \frac{\gamma + 1}{8} \left[2 + \frac{\gamma + 1}{4} (6\gamma - 1) \, \alpha^2 + \frac{1}{6} \left(\frac{\gamma + 1}{8} \right)^2 (544\gamma^2 \right.$$

$$\left. - 151\gamma - 33) \, \alpha^4 \right] \alpha, \tag{29}$$

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and $r_s = 1$ at the throat wall. The axial position of the intersection of the critical line and the throat wall can be found from the requirements that

$$(1 + u)^2 + v^2 = 1 \tag{30}$$

at $x = x_s$, $r = r_s$. Thus,

$$x_s = \frac{-P + \sqrt{P^2 - 4\ NQ}}{2N}, \tag{31}$$

where

$$N = \left[1 + \frac{\gamma + 1}{4}\ (6\gamma^2 - \gamma - 1)\ r_s^2\right]\alpha, \tag{32}$$

$$P = 2 + \gamma\ (\gamma + 1)\ \alpha^2 r_s^2 + 2\left(\frac{\gamma + 1}{8}\right)^2 (36\gamma^2 - 3\gamma - 7)\ \alpha^4 r_s^4, \tag{33}$$

and

$$Q = \frac{\gamma + 1}{8}\left[4 + \frac{\gamma + 1}{4}\ (6\gamma + 1)\ \alpha^2 r_s^2 + \frac{2}{3}\left(\frac{\gamma + 1}{8}\right)^2 (112\gamma^2 - \gamma - 9)\ \alpha^4 r_s^4\right]\alpha r_s^2. \tag{34}$$

The radial position of this point is then found from

$$r_s = 1 + \rho_s - \sqrt{\rho_s^2 - (x_s - \epsilon)^2}. \tag{35}$$

Now the system of equations that must be solved [(19), (25), (31), (35)] are interdependent and since equations (19) and (25) cannot be solved explicitly for the necessary roots, it is necessary to resort to a numerical iteration procedure to obtain the solution.

After the numerical solution of the above set of equations is completed, lines of constant velocity are determined from

$$(1 + u)^2 + v^2 = M^{*2}. \tag{36}$$

When the appropriate quantities are substituted into equation (36), the geometric position of the constant velocity line is

$$x = \frac{-P_1 + \sqrt{P_1^2 - 4N_1 Q_1}}{2N_1} - \epsilon, \tag{37}$$

where

$$N_1 = \alpha^2 \left[1 + \frac{\gamma + 1}{4} (6\gamma^2 - \gamma - 1) r^2 \right], \tag{38}$$

$$P_1 = \alpha \left[2 + \gamma(\gamma + 1) \alpha^2 r^2 + 2 \left(\frac{\gamma + 1}{8} \right)^2 (36\gamma^2 - 3\gamma - 7)\alpha^4 r^4 \right], \tag{39}$$

and

$$Q_1 = 1 - M^{*2} + \frac{\gamma + 1}{8} \left[4 + \frac{\gamma + 1}{4} (6\gamma + 1) \alpha^2 r^2 + \frac{2}{3} \left(\frac{\gamma + 1}{8} \right)^2 (112\gamma^2 - \gamma - 9) \alpha^4 r^4 \right] \alpha r^2. \tag{40}$$

One of the reasons for computing constant velocity lines is to use the data as start-line data for a method of characteristics solution of the downstream supersonic flow field. For all values of ρ_s and γ where constant velocity lines were computed, it was found that the slope of all constant velocity lines down to the throat section was less than the Mach angle at points near the nozzle wall. This makes the constant velocity lines unsuitable for starting lines since this makes computed points fall behind the start-line points. Since many nozzles have different radii

of curvature upstream and downstream of the throat, it is often desirable
to have the start line begin at the throat section wall. It was found
that an arbitrary parabola that passes through the throat wall could be
used successfully as a start line. Such a parabola can be defined as

$$X = K(1 - r^2),$$ (41)

where

$$K = \frac{M^*_{r=0} - 1}{\alpha} - \epsilon = K_1 - \epsilon.$$ (42)

For this parabola to be a valid start line, we must have

$$\frac{1}{2K} > \tan \mu_{r=1}$$ (43)

or

$$\frac{\alpha}{2(M^*_{r=0} - 1 - \alpha\epsilon)} > \sqrt{\frac{1 - \frac{\gamma - 1}{\gamma + 1} M^*_{r=1}}{M^{*2}_{r=1} - 1}},$$ (44)

where $M^*_{r=1}$ is the critical velocity at the throat wall and is given in
Table 4. Therefore, the critical velocity given on this arbitrary
parabola at the axis must satisfy the inequality of (44) if no computed
points are to fall on or behind the start line. The critical velocity
on the axis must always be smaller than the critical velocity at the
throat wall for the inequality of (44) to hold. With $M^*_{r=0}$ chosen the
geometry of the parabola is given by equation (41) and the velocity com-
ponents are given by

$$u = \alpha K_1 + \xi_1 r^2 + \xi_2 r^4 + \xi_3 r^6,$$ (45)

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$$\xi_1 = \left[-K + \alpha \frac{\gamma + 1}{4} \left\{ 1 + (2\gamma - 1) \alpha K_1 + 3\gamma(\gamma - 1) \alpha^2 K_1{}^2 \right\} \right] \alpha, \tag{46}$$

$$\xi_2 = \frac{\gamma + 1}{8} \left[-2K \left\{ 2\gamma - 1 + 6\gamma(\gamma - 1) \alpha K_1 \right\} + \alpha \frac{\gamma + 1}{8} \left\{ 6\gamma - 1 \right.\right.$$

$$\left.\left. + (36\gamma^2 - 19\gamma - 4) \alpha K_1 \right\} \right] \alpha^3, \tag{47}$$

$$\xi_3 = \frac{\gamma + 1}{8} \left[6\gamma(\gamma - 1) K^2 - \frac{\gamma + 1}{8} (36\gamma^2 - 19\gamma - 4) K\alpha \right.$$

$$\left. + \frac{2}{3} \left(\frac{\gamma + 1}{8} \right)^2 (56\gamma^2 - 20\gamma - 3) \alpha^2 \right] \alpha^4, \tag{48}$$

and

$$v = \eta_1 r + \eta_2 r^3 + \eta_3 r^5 + \eta_4 r^7, \tag{49}$$

where

$$\eta_1 = \frac{\gamma + 1}{4} \left[2 + (2\gamma - 1) \alpha K_1 + 2\gamma(\gamma - 1) \alpha^2 K_1{}^2 \right] \alpha^2 K_1, \tag{50}$$

$$\eta_2 = \frac{\gamma + 1}{4} \left[-2K \left\{ 1 + (2\gamma - 1) \alpha K_1 + 3\gamma (\gamma - 1) \alpha^2 K_1{}^2 \right\} \right.$$

$$\left. + \frac{\gamma + 1}{8} \alpha \left\{ 2 + 2(6\gamma - 1) \alpha K_1 + (36\gamma^2 - 19\gamma - 4) \alpha^2 K_1{}^2 \right\} \right] \alpha^2, \tag{51}$$

$$\eta_3 = \frac{\gamma + 1}{4} \left[K^2 \left\{ 2\gamma - 1 + 6\gamma(\gamma - 1) \alpha K_1 \right\} - \frac{\gamma + 1}{4} \alpha K \left\{ 6\gamma - 1 \right. \right.$$

$$\left. + (36\gamma^2 - 19\gamma - 4) \alpha K_1 \right\} + 2 \left(\frac{\gamma + 1}{8} \right)^2 \alpha^2 \left\{ 6\gamma - 1 \right.$$

$$\left. \left. + (56\gamma^2 - 20\gamma - 3) \alpha K_1 \right\} \right] \alpha^3 , \tag{52}$$

$$\eta_4 + \frac{\gamma + 1}{4} \left[-K \left\{ 2\gamma (\gamma - 1) K^2 - \frac{\gamma + 1}{8} (36\gamma^2 - 19\gamma - 4) K\alpha \right. \right.$$

$$\left. + 2 \left(\frac{\gamma + 1}{8} \right)^2 (56\gamma^2 - 20\gamma - 3) \alpha^2 \right\} + \frac{1}{6} \left(\frac{\gamma + 1}{8} \right)^3 (544\gamma^2$$

$$\left. - 151\gamma - 33) \alpha^3 \right] \alpha^4 . \tag{53}$$

Desired final results are given by

$$M^* = \sqrt{(1 + u)^2 + v^2} \tag{54}$$

$$\theta = \tan^{-1} \frac{v}{1 + u} . \tag{55}$$

IV. NUMERICAL SOLUTION AND RESULTS

It is possible to find a numerical iteration technique for the solution of the foregoing sets of equations that closes satisfactorily. Equation (19) for α was solved by Newton's root method. This equation appears to have only one real, positive root for typical values of ρ_s and γ and the derivative is regular. Equation (25) for ϵ has only one real negative root and its derivative is regular so that it is possible to solve this equation by Newton's root method. The iteration procedure used to overcome the interdependence of the four equations is outlined in the following procedure:

Step 1: Solve equation (19) for α with $x_s = 0$, $r_s = 1$.

Step 2: Solve equation (25) for ϵ, using α from step 1.

Step 3: Solve equation (31) for x_s, using α from step 1 and let

$$r_s = 1 + \rho_s - \sqrt{\rho_s^2 - \epsilon^2} \; .$$

Step 4: Solve equation (35) for r_s, using x_s from step 3 and ϵ from step 2.

Iteration

Step 5: Repeat step 1, using x_s from step 3 and r_s from step 4.

Step 6: Repeat step 2, using α from step 5.

Step 7: Repeat step 3, using α from step 5 and r_s from step 4.

Step 8: Repeat step 4, using x_s from step 7 and ϵ from step 6.

The iteration steps are repeated until two consecutive values of α agree to the desired tolerance.

Results of a parametric series of solutions of the equations are given in Tables 1 through 4 for various values of γ and ρ_s. For the higher values of γ and lower values of ρ_s, imaginary values of x_s were obtained from equation (31), and it was not possible to obtain a numerical solution for these cases. There seems to be no physical reason for the critical line not to reach the nozzle wall for these conditions. It appears, therefore, that the truncated series used to derive the equations are not adequate for these particular parametric combinations.

Equation (19) for α is an alternating series due to the fact that x_s is negative; and since the general term cannot be derived, it is impossible to prove that it is convergent. For small values of α, it has been shown numerically that the final terms used in equation (19) are insignificant, and it can be assumed that the series is closed. However, for larger values of α, the final term of equation (19) is significant, compared to the first term, and the solutions for large values of α are not closed.
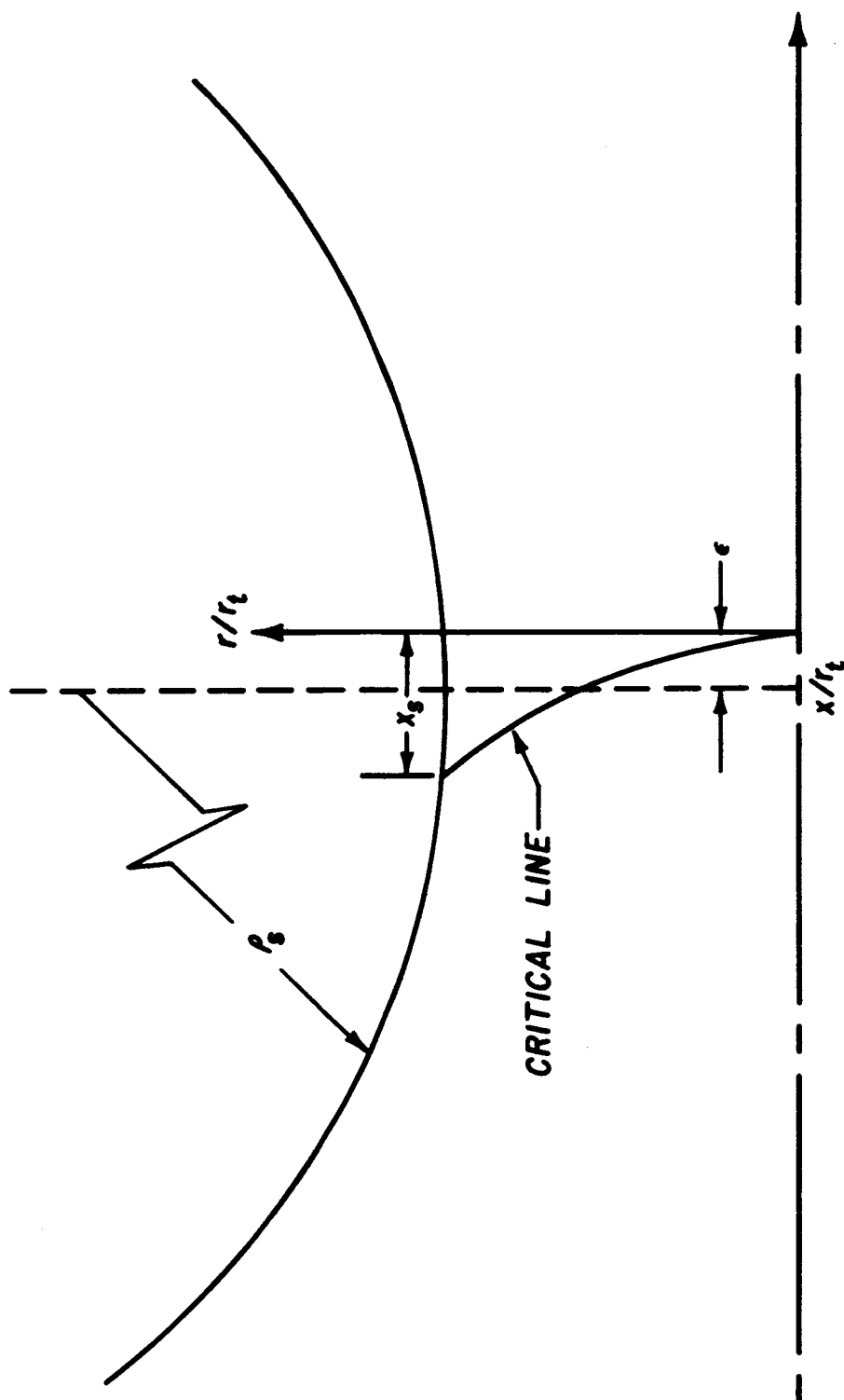
GEOMETRY OF THE PROBLEM

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TABLE 1

α TABLE

ρ_g/γ	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40
1.0	.761866	.759001	.756390	.754087	.752148	.750655	.749756	.749671	.750997							
1.25	.710649	.708254	.706178	.704484	.703276	.702698	.703019	.704743	.709161							
1.50	.667224	.664978	.663042	.661478	.660396	.659960	.660474	.662591	.668461							
1.75	.630384	.628152	.626180	.624530	.623280	.625674	.626618	.623929	.627920							
2.0	.598878	.596613	.594566	.592782	.591309	.590246	.589733	.590055	.591861							
2.5	.547838	.545538	.543390	.541405	.539614	.538054	.536787	.535889	.535520	.535975	.538065					
3.0	.508109	.505830	.503655	.501596	.499666	.497887	.496274	.494863	.493703	.492881	.492517	.492903	.494816			
4.0	.449543	.447384	.445291	.443266	.441312	.439445	.437654	.435954	.434355	.432871	.431520	.430339	.429349	.428620	.428271	.428522
5.0	.407725	.405713	.403748	.401832	.399967	.398153	.396393	.394689	.393044	.391464	.389963	.388529	.387181	.385929	.384789	.383793
6.0	.375883	.374005	.372164	.370373	.368612	.366891	.365211	.363571	.361975	.360423	.358917	.357459	.356053	.354702	.353422	.352201
7.0	.350573	.348813	.347087	.345392	.343730	.342101	.340515	.338954	.337426	.335934	.334476	.333055	.331671	.330326	.329023	.327762
8.0	.329808	.328152	.326524	.324924	.323353	.321811	.320296	.318810	.317352	.315922	.314535	.313166	.311827	.310519	.309242	.307998
9.0	.312366	.310798	.309256	.307740	.306249	.304784	.303344	.301928	.300538	.299172	.297831	.296516	.295225	.293974	.292737	.291527
10.0	.297442	.295951	.294484	.293041	.291621	.290224	.288850	.287498	.286169	.284863	.283578	.282316	.281076	.279859	.278663	.277491
12.0	.273082	.271719	.270377	.269055	.267753	.266471	.265209	.263967	.262744	.261539	.260354	.259187	.258038	.256908	.255797	.254703
15.0	.245684	.244463	.243258	.242072	.240923	.239773	.238641	.237525	.236426	.235342	.234275	.233223	.232186	.231164	.230157	.229165
20.0	.214064	.213009	.211970	.210945	.209934	.208938	.207955	.206986	.206031	.205088	.204179	.203265	.202363	.201473	.200595	.199729

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TABLE 2

€ TABLE

Pg/γ	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40
1.0	-.218180	-.220018	-.221910	-.223897	-.225995	-.228237	-.230682	-.233414	-.236662							
1.25	-.201755	-.203502	-.205341	-.207297	-.209411	-.211737	-.214379	-.217525	-.221658							
1.50	-.188003	-.189598	-.191282	-.193081	-.195035	-.197207	-.199708	-.202781	-.207208							
1.75	-.176478	-.177911	-.179416	-.181018	-.182745	-.184650	-.186815	-.189422	-.192984							
2.0	-.166733	-.168017	-.169359	-.170775	-.172287	-.173929	-.175753	-.177865	-.180498							
2.5	-.151183	-.152239	-.153330	-.154462	-.155647	-.156899	-.158241	-.159702	-.161341	-.163265	-.165764					
3.0	-.139290	-.140190	-.141111	-.142057	-.143033	-.144049	-.145109	-.146229	-.147426	-.148733	-.150195	-.151917	-.154179			
4.0	-.122100	-.122811	-.123532	-.124262	-.125004	-.125763	-.126537	-.127331	-.128149	-.128998	-.129883	-.130819	-.131815	-.132896	-.134108	-.135530
5.0	-.110070	-.110674	-.111282	-.111895	-.112513	-.113137	-.113769	-.114409	-.115060	-.115723	-.116403	-.117099	-.117816	-.118559	-.119333	-.120152
6.0	-.101041	-.101575	-.102110	-.102650	-.103190	-.103734	-.104281	-.104832	-.105388	-.105950	-.106519	-.107097	-.107683	-.108281	-.108896	-.109524
7.0	-.093940	-.094424	-.094908	-.095394	-.095880	-.096367	-.096860	-.097352	-.097846	-.098344	-.098845	-.099350	-.099860	-.100376	-.100899	-.101430
8.0	-.088161	-.088608	-.089054	-.089501	-.089947	-.090395	-.090842	-.091291	-.091741	-.092192	-.092650	-.093106	-.093564	-.094026	-.094491	-.094961
9.0	-.083333	-.083755	-.084172	-.084588	-.085004	-.085420	-.085836	-.086252	-.086668	-.087086	-.087504	-.087923	-.088343	-.088769	-.089194	-.089621
10.0	-.079237	-.079625	-.080018	-.080409	-.080800	-.081191	-.081581	-.081972	-.082362	-.082752	-.083142	-.083533	-.083925	-.084317	-.084710	-.085105
12.0	-.072574	-.072930	-.073286	-.073635	-.073988	-.074340	-.074691	-.075042	-.075392	-.075742	-.076092	-.076441	-.076790	-.077139	-.077488	-.077837
15.0	-.065135	-.065450	-.065765	-.066078	-.066396	-.066708	-.067019	-.067325	-.067634	-.067942	-.068250	-.068557	-.068863	-.069169	-.069474	-.069779
20.0	-.056613	-.056885	-.057156	-.057426	-.057694	-.057962	-.058229	-.058494	-.058759	-.059023	-.059292	-.059554	-.059817	-.060078	-.060338	-.060598

TABLE 3

X$_S$ TABLE

s/γ	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40
1.0	-.514205	-.521789	-.530328	-.540110	-.551428	-.564761	-.581031	-.601852	-.632320							
1.25	-.475602	-.483657	-.492784	-.503236	-.515428	-.529966	-.547941	-.571487	-.606237							
1.50	-.442470	-.450310	-.459174	-.469338	-.481227	-.495509	-.513404	-.537574	-.576844							
1.75	-.413584	-.420822	-.428953	-.438216	-.448949	-.461713	-.477472	-.498269	-.530145							
2.0	-.388499	-.395037	-.402317	-.410521	-.419904	-.430857	-.444028	-.460634	-.483556							
2.5	-.347772	-.353056	-.358842	-.365222	-.372331	-.380352	-.389560	-.400370	-.413514	-.430449	-.455118					
3.0	-.316513	-.320866	-.325563	-.330661	-.336233	-.342378	-.349215	-.356925	-.365770	-.376175	-.388843	-.405269	-.429640			
4.0	-.271895	-.275070	-.278435	-.282012	-.285828	-.289926	-.294336	-.299113	-.304326	-.310060	-.316436	-.323630	-.331871	-.341557	-.353391	-.368802
5.0	-.241415	-.243914	-.246532	-.249282	-.252177	-.255233	-.258469	-.261908	-.265576	-.269508	-.273753	-.278346	-.283360	-.288883	-.295033	-.301987
6.0	-.219060	-.221128	-.223278	-.225525	-.227864	-.230309	-.232872	-.235564	-.238400	-.241396	-.244571	-.247948	-.251555	-.255427	-.259616	-.264159
7.0	-.201828	-.203601	-.205435	-.207334	-.209303	-.211349	-.213485	-.215705	-.218024	-.220452	-.222999	-.225678	-.228504	-.231494	-.234669	-.238055
8.0	-.188042	-.189602	-.191208	-.192864	-.194573	-.196340	-.198168	-.200064	-.202031	-.204077	-.206220	-.208448	-.210778	-.213222	-.215790	-.218498
9.0	-.176703	-.178100	-.179535	-.181008	-.182524	-.184085	-.185694	-.187355	-.189071	-.190848	-.192689	-.194600	-.196587	-.198669	-.200832	-.203095
0.0	-.167169	-.168440	-.169740	-.171072	-.172439	-.173841	-.175283	-.176766	-.178293	-.179867	-.181493	-.183173	-.184913	-.186716	-.188588	-.190535
2.0	-.151933	-.153016	-.154121	-.155248	-.156399	-.157575	-.158777	-.160008	-.161269	-.162563	-.163890	-.165254	-.166656	-.168100	-.169589	-.171125
5.0	-.135249	-.136149	-.137062	-.137989	-.138944	-.139905	-.140883	-.141878	-.142893	-.143928	-.144984	-.146063	-.147165	-.148293	-.149447	-.150630
0.0	-.116545	-.117266	-.117994	-.118731	-.119476	-.120229	-.120993	-.121766	-.122550	-.123344	-.124164	-.124985	-.125818	-.126665	-.127527	-.128403

TABLE 4

M*_R = 1.0 TABLE

g/γ	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40
.0	1.23933	1.24086	1.24260	1.24461	1.24695	1.24972	1.25310	1.25734	1.26315							
.25	1.19575	1.19712	1.19871	1.20061	1.20289	1.20572	1.20933	1.21425	1.22174							
.50	1.16432	1.16539	1.16665	1.16815	1.16100	1.17232	1.17535	1.17966	1.18702							
.75	1.14108	1.14187	1.14280	1.14392	1.14528	1.14996	1.15312	1.15223	1.15717							
.0	1.12341	1.12398	1.12466	1.12546	1.12644	1.12763	1.12914	1.13115	1.13407							
.5	1.09854	1.09885	1.09922	1.09964	1.10015	1.10075	1.10148	1.10239	1.10355	1.10513	1.10756					
.0	1.08201	1.08218	1.08239	1.08262	1.08290	1.08323	1.08362	1.08408	1.08465	1.08535	1.08624	1.08747	1.08939			
.0	1.06146	1.06152	1.06159	1.06168	1.06178	1.06190	1.06204	1.06220	1.06238	1.06260	1.06286	1.06317	1.06354	1.06400	1.06459	1.06540
.0	1.04919	1.04922	1.04925	1.04928	1.04932	1.04937	1.04943	1.04950	1.04958	1.04967	1.04977	1.04990	1.05004	1.05020	1.05039	1.05063
.0	1.04103	1.04104	1.04105	1.04107	1.04108	1.04111	1.04114	1.04117	1.04121	1.04125	1.04130	1.04136	1.04142	1.04150	1.04159	1.04169
.0	1.03521	1.03521	1.03522	1.03522	1.03523	1.03524	1.03525	1.03527	1.03529	1.03531	1.03534	1.03537	1.03540	1.03544	1.03549	1.03554
.0	1.03084	1.03084	1.03084	1.03084	1.03084	1.03085	1.03085	1.03086	1.03087	1.03088	1.03090	1.03092	1.03093	1.03096	1.03098	1.03101
.0	1.02744	1.02744	1.02744	1.02743	1.02743	1.02744	1.02744	1.02744	1.02745	1.02745	1.02746	1.02747	1.02748	1.02750	1.02751	1.02753
.0	1.02471	1.02471	1.02471	1.02471	1.02471	1.02471	1.02471	1.02471	1.02471	1.02471	1.02472	1.02472	1.02473	1.02474	1.02475	1.02476
.0	1.02062	1.02062	1.02062	1.02062	1.02062	1.02061	1.02061	1.02061	1.02061	1.02061	1.02061	1.02061	1.02062	1.02062	1.02062	1.02063
.0	1.01653	1.01652	1.01652	1.01652	1.01652	1.01651	1.01651	1.01651	1.01651	1.01651	1.01651	1.01651	1.01651	1.01651	1.01651	1.01651
.0	1.01242	1.01241	1.01241	1.01241	1.01241	1.01241	1.01241	1.01240	1.01240	1.01240	1.01240	1.01240	1.01240	1.01240	1.01240	1.01240

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CALCULATION OF TRANSONIC NOZZLE FLOW

By Joseph L. Sims

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Werner K. Dahm
Chief, Aerodynamics Division

_____
E. D. Geissler
Director, Aero-Astrodynamics Laboratory

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